Final Exam Review - Day 1

Ex: $\lim_{x \to 3} \frac{x^2+7}{x+1} = \frac{3^2+7}{3+1} = \frac{16}{4} = \boxed{4}$

Ex: Suppose $\lim_{x \to 1} f(x) = 3$ and $\lim_{x \to 1} g(x) = -7$

Determine $\lim_{x \to 1} \left(x \cdot f(x) + \frac{g(x)}{x+1} \right)$

$$= \left(\lim_{x \to 1} x\right)\left(\lim_{x \to 1} f(x)\right) + \frac{\lim_{x \to 1} g(x)}{\lim_{x \to 1} x+1}$$

$$= 1 \cdot 3 + \frac{-7}{2}$$

$$= 3 - \frac{7}{2}$$

$$= \boxed{\frac{-1}{2}}$$

Ex: $\lim_{x \to 5} \dfrac{x^2 - x - 20}{x - 5}$

"$\frac{0}{0}$" do more work!

$= \lim_{x \to 5} \dfrac{(x-5)(x+4)}{x-5}$

$= \lim_{x \to 5} x + 4 = 5 + 4 = \boxed{9}$

Ex: $\lim_{x \to \infty} \dfrac{(3x-1)^2}{5x^2+8} = \lim_{x \to \infty} \dfrac{9x^2}{5x^2}$

$= \lim_{x \to \infty} \dfrac{9}{5} = \boxed{\dfrac{9}{5}}$

Ex: Use the definition of the derivative to find f' for $f(x) = 3x^2 + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)^2 + 1 - (3x^2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2}{h} = \lim_{h \to 0} 6x + 3h$$

$$= 6x + 3(0) = \boxed{6x}$$

Ex: let $f(x) = \frac{6x}{\sqrt{x}}$. find $f'(x)$

rewrite and use product rule!

$$f(x) = 6x \cdot x^{-1/2}$$

$$f'(x) = (6x)\left(-\tfrac{1}{2}x^{-3/2}\right) + 6\left(x^{-1/2}\right)$$

$$= \frac{-6x}{2x^{3/2}} + \frac{6}{x^{1/2}} = \boxed{\frac{-3x}{x^{3/2}} + \frac{6}{x^{1/2}}}$$

Ex: let $f(x) = \sqrt{x^2+3}$ find $f'(x)$.

chain rule!

$$f(x) = (x^2+3)^{1/2}$$

thus $\boxed{f'(x) = \frac{1}{2}(x^2+3)^{-1/2}(2x)}$

Ex: find the derivative of

$$f(x)=\int_3^x (9t+15)\,dt$$

FTC - part 1

$$f'(x)=9x+15$$

Ex: Suppose the height in feet of an object above ground at time t (in seconds) is given by $h(t) = -16t^2 + 15t + 300$.

Find the acceleration of the object after 3 seconds.

$h'(t) = -32t + 15$ velocity

$h''(t) = -32$ acceleration

then $h''(3) = \boxed{-32 \text{ units/sec}^2}$

Ex: How many years will it take for an investment to double in value if the interest is compounded continuously at a rate of 5%?

$$P(t) = P_0 e^{rt}$$

$$2P_0 = P_0 e^{.05t}$$

$$2 = e^{.05t}$$

$$\ln 2 = \ln e^{.05t}$$

$$\ln 2 = .05t$$

$$t = \frac{\ln 2}{.05} \approx \boxed{13.86 \text{ years}}$$